Genuine twist-3 contributions to the generalized parton distributions from instantons

D.V. Kiptily[1], M.V. Polyakov[1,2]

[1] Institut für Theoretische Physik II, Ruhr–Universität Bochum, 44780 Bochum, Germany
[2] Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188350, Russia

Abstract. The genuine twist-3 quark–gluon ($\bar{q}Gq$) contributions to the generalized parton distributions (GPDs) are estimated in the model of the instanton vacuum. These twist-3 effects are found to be parametrically suppressed relative to the "kinematical" twist-3 ones due to the small packing fraction of the instanton vacuum. We derived exact sum rules for the twist-3 GPDs.

1 Twist-3 DVCS amplitude

Deeply virtual Compton scattering (DVCS) [1,2] has been a subject of rather intensive investigations during the last few years (for a detailed recent review of DVCS see [3]). It is a two-photon process:

$$\gamma^*(q) + N(P) \to \gamma'(q') + N'(P') , \tag{1}$$

in which the initial virtual photon γ^*, being produced by a lepton, interacts with a parton of the initial nucleon N, the final real photon γ' is radiated by the parton, and the nucleon N' in the final state is formed by the active parton and the rest of the initial nucleon N. The reaction is considered in the Bjorken limit, when the virtuality of the initial photon $Q^2 = -q^2 \gg P^2$ is large, the Bjorken variable $x_{\text{Bj}} = Q^2/2(P \cdot q)$ is fixed and the momentum transfer squared is small: $(P' - P)^2 \ll Q^2$. The first experimental results on DVCS can be found in [4–7].

We will work in the infinite momentum frame defined by the light-cone vectors $n^\mu = 1/(\bar{P}_+ \sqrt{2})(1,0,0,-1)$ and $p^\mu = \bar{P}_+/\sqrt{2}(1,0,0,1)$, with $\bar{P}^\mu = (P + P')^\mu/2$ being the average hadron momentum (note that $n^2 = p^2 = 0$ and $n \cdot p = n \cdot \bar{P} = 1$). Let us define the momentum transfer as $\Delta^\mu = (P' - P)^\mu$ and its square as $t = (P' - P)^2$. In this paper we shall use a kinematical variable ξ, analogous to the Bjorken one, defined as $\xi = -(n \cdot \Delta)/2$. A more detailed description of the DVCS kinematics can be found, e.g., in [3].

To leading order in the electromagnetic coupling constant, the amplitude of DVCS is proportional to the hadronic tensor $H_{\mu\nu}$,

$$H_{\mu\nu} = -\mathrm{i} \int \mathrm{d}^4x \, e^{-\mathrm{i}qx} \langle P'|T J_\mu(x) J_\nu(0)|P \rangle . \tag{2}$$

To order $(1/Q)^0$, it can be expressed through the non-diagonal hadron matrix elements $\mathcal{F}_\alpha$ and $\widetilde{\mathcal{F}}_\alpha$ of gauge-invariant light-cone bilocal operators:

$$H_{\mu\nu} = \frac{1}{2} \left(-g^\perp_{\mu\nu} \right) \int_{-1}^1 \mathrm{d}x \, C_+(x,\xi) \, n^\alpha \mathcal{F}_\alpha(x,\xi,t) \tag{3}$$
$$- \frac{\mathrm{i}}{2} \varepsilon^\perp_{\mu\nu} \int_{-1}^1 \mathrm{d}x \, C_-(x,\xi) \, n^\alpha \widetilde{\mathcal{F}}_\alpha(x,\xi,t) + \mathcal{O}\left(\frac{1}{Q}\right) ,$$

where $\mathcal{F}_\alpha$ and $\widetilde{\mathcal{F}}_\alpha$ are defined as

$$\mathcal{F}_\alpha(x,\xi,t) \tag{4}$$
$$= \int_{-\infty}^{+\infty} \frac{\mathrm{d}\lambda}{2\pi} \, e^{-\mathrm{i}\lambda x} \left\langle P', S' \left| \bar{\psi}\left(\frac{\lambda}{2}n\right) \gamma_\alpha \psi\left(-\frac{\lambda}{2}n\right) \right| P, S \right\rangle ,$$

$$\widetilde{\mathcal{F}}_\alpha(x,\xi,t) \tag{5}$$
$$= \int_{-\infty}^{+\infty} \frac{\mathrm{d}\lambda}{2\pi} \, e^{-\mathrm{i}\lambda x} \left\langle P', S' \left| \bar{\psi}\left(\frac{\lambda}{2}n\right) \gamma_\alpha \gamma_5 \psi\left(-\frac{\lambda}{2}n\right) \right| P, S \right\rangle$$

(let us note, that here we do not write explicitly the phase factors which make the bilocal operators gauge-invariant). The coefficient functions $C_\pm(x,\xi)$ are

$$C_\pm(x,\xi) = \frac{1}{x - \xi + \mathrm{i}\varepsilon} \pm \frac{1}{x + \xi - \mathrm{i}\varepsilon} , \tag{6}$$

and the transverse tensors $-g^\perp_{\mu\nu}$ and $\varepsilon^\perp_{\mu\nu}$ are

$$-g^\perp_{\mu\nu} = -g_{\mu\nu} + n_\mu p_\nu + n_\nu p_\mu , \qquad \varepsilon^\perp_{\mu\nu} = \varepsilon_{\mu\nu\lambda\sigma} n^\lambda p^\sigma . \tag{7}$$

In general, the functions $\mathcal{F}_\alpha$ and $\widetilde{\mathcal{F}}_\alpha$ contain both the purely quark and mixed quark–gluon contributions. However, in the considered case, the quark–gluon contribution vanishes due to contraction with the light-cone vector n^α. Therefore

the scalar products $n^\alpha \mathcal{F}_\alpha$ and $n^\alpha \widetilde{\mathcal{F}}_\alpha$ can be parameterized in terms of four functions $H_q(x,\xi,t)$, $E_q(x,\xi,t)$ and $\widetilde{H}_q(x,\xi,t)$, $\widetilde{E}_q(x,\xi,t)$ in the following way [2]:

$$n^\alpha \mathcal{F}_\alpha = H_q(x,\xi,t)\, \bar{N}(P',S')\hat{n}N(P,S)$$

$$+ E_q(x,\xi,t)\, \bar{N}(P',S')\frac{\mathrm{i}\sigma^{\mu\nu}n_\mu \Delta_\nu}{2M}N(P,S)\,, \quad (8)$$

$$n^\alpha \widetilde{\mathcal{F}}_\alpha = \widetilde{H}_q(x,\xi,t)\, \bar{N}(P',S')\hat{n}\gamma_5 N(P,S)$$

$$+ \widetilde{E}_q(x,\xi,t)\, \bar{N}(P',S')\frac{\gamma_5 \Delta \cdot n}{2M}N(P,S)\,. \quad (9)$$

Here $N(P,S)$ is the Dirac bispinor of a hadron (a proton in our case), M is the hadron mass, and we use the notation $\hat{x} = x^\mu \gamma_\mu$ throughout the paper. The functions H, E and $\widetilde{H}$, $\widetilde{E}$ are the generalized parton distributions (GPD) (for a review, see [8–10]). In the forward limit $P' = P$ the distributions H_q and $\widetilde{H}_q$ coincide with the usual parton densities $q(x)$ and $\Delta q(x)$. The distributions E_q and $\widetilde{E}_q$ are completely new ones. Thus the investigation of DVCS allows us to get additional information about the structure of hadrons. In particular, the GPDs contain information about the hadron spin distribution among the partons of the hadron [2,11].

The amplitude of DVCS has corrections of two types: firstly, loop corrections in powers of the electromagnetic and strong coupling constants and, secondly, higher-twist corrections in powers of $1/Q$. We will not consider the loop corrections in this paper. Making allowance for the first correction in powers of $1/Q$, the hadronic tensor $H_{\mu\nu}$ takes the following form [12,13] (for the pion target see [14,15]):

$$H_{\mu\nu}$$

$$= \frac{1}{2}\left[(-g_{\mu\nu}^\perp) - \frac{\Delta_\mu^\perp \bar{P}_\nu}{(\bar{P}\cdot q')}\right]$$

$$\times \int_{-1}^{1} \mathrm{d}x\, C_+(x,\xi)\, n^\alpha \mathcal{F}_\alpha(x,\xi,t)$$

$$+ \frac{\mathrm{i}}{2}\varepsilon_\mu^{\perp\lambda}\left[(-g_{\lambda\nu}^\perp) - \frac{\Delta_\lambda^\perp \bar{P}_\nu}{(\bar{P}\cdot q')}\right]$$

$$\times \int_{-1}^{1} \mathrm{d}x\, C_-(x,\xi)\, n^\alpha \widetilde{\mathcal{F}}_\alpha(x,\xi,t)$$

$$- \frac{(q + 4\xi\bar{P})_\mu}{2\,(\bar{P}\cdot q')}\left[(-g_{\lambda\nu}^\perp) - \frac{\Delta_\lambda^\perp \bar{P}_\nu}{(\bar{P}\cdot q')}\right]$$

$$\times \int_{-1}^{1} \mathrm{d}x\, \Big\{\mathcal{F}_\perp^\lambda(x,\xi,t)\, C_+(x,\xi)$$

$$- \mathrm{i}\varepsilon_\perp^{\lambda\sigma}\widetilde{\mathcal{F}}_\sigma^\perp(x,\xi,t)\, C_-(x,\xi)\Big\}$$

$$+ \mathcal{O}\left(\frac{1}{Q^2}\right). \quad (10)$$

Note that $H_{\mu\nu}$ satisfies the transversity conditions

$$q^\mu H_{\mu\nu} = 0\,, \quad H_{\mu\nu}(q')^\nu = 0\,. \quad (11)$$

We see that the correction contains a term proportional to the transverse part of the vector functions $\mathcal{F}_\alpha$ and $\widetilde{\mathcal{F}}_\alpha$. This means that, in comparison to the leading order, to order $1/Q$ we deal with corrections of two types: those, called "kinematical", associated with purely quark operators and "genuine twist-3" ones originating from the quark–gluon operators, which have the form $\bar{\psi}G\psi$, where ψ is the operator of the quark field and G denotes the operator of the gluon field strength tensor. The "kinematical" contribution can be expressed through the twist-2 quark GPDs H, E and $\widetilde{H}$, $\widetilde{E}$ [13–17].

In the case of polarized DIS the quark–gluon correction is supposed to be small relative to the "kinematical" one. This assumption is known as Wandzura–Wilczek approximation [18]. The experimental data [19–21] point at its reliability. It is assumed that the same approximation is valid in the case of DVCS. The aim of this paper is to test this hypothesis in the model of the instanton vacuum [22].

A method based on the model of the instanton vacuum which allows one to calculate hadronic matrix elements of the quark–gluon operators, was suggested [23]. In the case of DIS it was shown [24,25] that the operators of twist-3 are of order $(\rho/R)^4 \ln(\rho/R)$, where ρ is the average size of the instantons and R is the average distance between instantons in the instanton medium. Thus, the matrix elements are parametrically small due to the packing fraction $\rho/R \sim 1/3$. In this paper we apply this analysis to the non-diagonal hadronic matrix elements of the quark–gluon operators in the framework of DVCS. We show that in the model of the instanton vacuum they are parametrically suppressed by the packing fraction of the instanton vacuum. In the forward limit we reproduce the results for the diagonal case, that is, for polarized DIS.

2 "Genuine twist-3" effects

Let us consider the following vector and axial-vector bilocal operators

$$V_\alpha(x,-x) = \bar{\psi}(x)\gamma_\alpha[x,-x]\psi(-x)\,, \quad (12)$$

$$A_\alpha(x,-x) = \bar{\psi}(x)\gamma_\alpha\gamma_5[x,-x]\psi(-x)\,. \quad (13)$$

Here

$$[x,y] = P\exp\left\{\mathrm{i}g\int_0^1 \mathrm{d}u\,(x-y)^\nu A_\nu(ux + (1-u)y)\right\} \quad (14)$$

is the phase factor ordered along a straight line connecting the points x and y. In what follows we will not write it explicitly, assuming its presence in all non-local gauge-invariant operators. In order to separate the "genuine twist-3" contributions to the operators (12) and (13) from the "kinematical" ones we shall use a technique elaborated in [13–17]. According to the result obtained by Balitsky and Braun [26,27] in the case of massless fermion operators there are the following operator identities ($\varepsilon_{0123} = +1$):

$$\bar{\psi}(x)\gamma_\alpha\psi(-x)$$

$$= \frac{\partial}{\partial x^\alpha} \int_0^1 du\, \bar\psi(ux)\hat{x}\psi(-ux)$$

$$-i\varepsilon_{\alpha\beta\lambda\sigma} \int_0^1 du\, ux^\beta \mathfrak{D}^\lambda \left[\bar\psi(ux)\gamma^\sigma\gamma_5\psi(-ux)\right]$$

$$+ \int_0^1 du\, \mathfrak{G}_\alpha(ux)\,, \tag{15}$$

$$\bar\psi(x)\gamma_\alpha\gamma_5\psi(-x)$$

$$= \frac{\partial}{\partial x^\alpha} \int_0^1 du\, \bar\psi(ux)\hat{x}\gamma_5\psi(-ux)$$

$$-i\varepsilon_{\alpha\beta\lambda\sigma} \int_0^1 du\, ux^\beta \mathfrak{D}^\lambda \left[\bar\psi(ux)\gamma^\sigma\psi(-ux)\right]$$

$$+ \int_0^1 du\, \mathfrak{G}_{5\alpha}(ux)\,, \tag{16}$$

where

$$\mathfrak{G}_\alpha(x) = g \int_{-1}^1 dv\, \bar\psi(x) \tag{17}$$

$$\times \left[\varepsilon_{\alpha\mu\nu\tau} G^{\mu\rho}(vx)x_\rho x^\nu \gamma^\tau \gamma_5 - ivG_{\alpha\rho}(vx)x^\rho \hat{x}\right] \psi(-x)\,,$$

$$\mathfrak{G}_{5\alpha}(x) = g \int_{-1}^1 dv\, \bar\psi(x) \tag{18}$$

$$\times \left[\varepsilon_{\alpha\mu\nu\tau} G^{\mu\rho}(vx)x_\rho x^\nu \gamma^\tau \gamma_5 - ivG_{\alpha\rho}(vx)x^\rho \hat{x}\right] \gamma_5\psi(-x)\,.$$

In these equations the derivative $\mathfrak{D}^\lambda$ is defined for any combination Γ of Dirac γ-matrices as

$$\mathfrak{D}^\lambda \left\{\bar\psi(x)\Gamma[x,-x]\psi(-x)\right\} \tag{19}$$

$$= \lim_{y\to 0} \frac{\partial}{\partial y_\lambda} \left\{\bar\psi(x+y)\Gamma[x+y,-x+y]\psi(-x+y)\right\}.$$

Later we will use the fact that the non-diagonal matrix element of (19) is proportional to the momentum transfer:

$$\left\langle P',S' \left| \mathfrak{D}^\lambda \left\{\bar\psi(x)\Gamma[x,-x]\psi(-x)\right\} \right| P,S \right\rangle \tag{20}$$

$$= i(P'-P)^\lambda \left\langle P',S' \left| \bar\psi(x)\Gamma[x,-x]\psi(-x) \right| P,S \right\rangle.$$

If we took the fermion mass into account, there would appear an additional mass term in the expression (16) for the axial-vector bilocal operator

$$+ 2im \int_0^1 du\, u\, \bar\psi(ux)i\sigma_{\alpha\beta}x^\beta \gamma_5\psi(-ux). \tag{21}$$

We would now like to express the operators (12) and (13) through the symmetrical ones:

$$V_{\text{sym}}(x,-x) = \bar\psi(x)\hat{x}[x,-x]\psi(-x)\,, \tag{22}$$

$$A_{\text{sym}}(x,-x) = \bar\psi(x)\hat{x}\gamma_5[x,-x]\psi(-x)\,. \tag{23}$$

These operators are "symmetrical" in the sense that on the light cone ($x^2 = 0$) they expand in series of symmetrical traceless local operators. In order to do so we should solve

the system of ordinary differential equations (15) and (16) with respect to $V_\alpha(x,-x)$ and $A_\alpha(x,-x)$ (see the appendix of [17] for details). The vector solution can be presented in the following form:

$$V_\alpha(x,-x) = V_\alpha^{\text{WW}}(x,-x) + V_\alpha^{\text{tw3}}(x,-x)\,, \tag{24}$$

where

$$V_\alpha^{\text{WW}}(x,-x)$$

$$= \frac{1}{2} \int_0^1 du \left\{ e^{(1-u)[(x\cdot\mathfrak{D})^2 - x^2\mathfrak{D}^2]^{1/2}} \right.$$

$$\left. + e^{-(1-u)[(x\cdot\mathfrak{D})^2 - x^2\mathfrak{D}^2]^{1/2}} \right\}$$

$$\times \left\{ \left[x_\alpha \mathfrak{D}^2 - (x\cdot\mathfrak{D})\mathfrak{D}_\alpha \right] \right.$$

$$\times \int_0^u dv\, v\, \bar\psi(vx)\hat{x}[vx,-vx]\psi(-vx)$$

$$+ \frac{\partial}{\partial x^\alpha} \left[\bar\psi(ux)\hat{x}[ux,-ux]\psi(-ux)\right]$$

$$- i\varepsilon_{\alpha\beta\lambda\sigma}x^\beta \tag{25}$$

$$\left. \times \mathfrak{D}^\lambda \frac{\partial}{\partial x_\sigma} \int_0^u dv\, \bar\psi(vx)\hat{x}\gamma_5[vx,-vx]\psi(-vx) \right\},$$

$$V_\alpha^{\text{tw3}}(x,-x)$$

$$= \frac{1}{2} \int_0^1 du \left\{ e^{(1-u)[(x\cdot\mathfrak{D})^2 - x^2\mathfrak{D}^2]^{1/2}} \right.$$

$$\left. + e^{-(1-u)[(x\cdot\mathfrak{D})^2 - x^2\mathfrak{D}^2]^{1/2}} \right\}$$

$$\times \left\{ -i\left[x^2\mathfrak{D}_\alpha - x_\alpha(x\cdot\mathfrak{D}) \right] \right.$$

$$\times \int_0^u dv\, v \int_{-v}^v dt\, \bar\psi(vx)[vx,tx]$$

$$\times gG_{\mu\nu}(tx)x^\mu\gamma^\nu[tx,-vx]\psi(-vx)$$

$$\left. + \mathfrak{G}_\alpha(ux) - i\varepsilon_{\alpha\beta\lambda\sigma}x^\beta\mathfrak{D}^\lambda \int_0^u dv\, \mathfrak{G}_5^\sigma(vx) \right\}. \tag{26}$$

Here V_α^{WW} is the "kinematical" (or Wandzura–Wilczek) contribution to the bilocal vector operator (12). The WW-contribution for the spin-$\frac{1}{2}$ target was discussed in detail in [13, 16]. The second term V_α^{tw3} in (24) is the "genuine twist-3" (quark–gluon) contribution to the vector operator. In the case $x^\alpha = -\kappa n^\alpha$ (24)–(26) reproduce the result of Belitsky and Müller [13]. It is of necessity to note that the "genuine twist-3" contribution to the scalar product $x^\alpha V_\alpha(x,-x)$ vanishes, with the first term of the "kinematical" contribution only surviving. In the case when $x^\alpha = \lambda n^\alpha/2$ this means that the quark–gluon contribution to the function $n^\alpha \mathcal{F}_\alpha$ vanishes, giving rise to the decomposition (8).

The corresponding expressions for the axial-vector operator (13) can be obtained by the following substitution

in (24)–(26):

$$\Gamma\psi \longrightarrow \Gamma\gamma_5\psi\,,$$

$$\mathfrak{G}_\alpha \longleftrightarrow \mathfrak{G}_{5\alpha}\,.$$

3 Diagonal nucleon matrix elements

Let us consider the polarized structure functions defined as integrals over diagonal matrix elements of string operators (for a review see [28, 29] and references therein):

$$\frac{1}{2M}\int\limits_{-\infty}^{+\infty}\frac{\mathrm{d}\lambda}{2\pi}\,\mathrm{e}^{-\mathrm{i}\lambda x}$$

$$\times \left\langle P,S \left| \bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right| P,S\right\rangle \qquad (27)$$

$$= g_1(x)(S\cdot n)p_\mu + g_\mathrm{T}(x)S_{\perp\mu} + g_3(x)M^2(S\cdot n)n_\mu\,,$$

$$\frac{1}{2M}\int\limits_{-\infty}^{+\infty}\frac{\mathrm{d}\lambda}{2\pi}\,\mathrm{e}^{-\mathrm{i}\lambda x}$$

$$\times \left\langle P,S \left| \bar{\psi}\left(\frac{\lambda n}{2}\right)\mathrm{i}\sigma_{\mu\nu}\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right| P,S\right\rangle$$

$$= h_1(x)\frac{1}{M}(S_{\perp\mu}p_\nu - S_{\perp\nu}p_\mu)$$

$$+ h_L(x)M(S\cdot n)(p_\mu n_\nu - p_\nu n_\mu)$$

$$+ h_3(x)M(S_{\perp\mu}n_\nu - S_{\perp\nu}n_\mu), \qquad (28)$$

where the spin 4-vector is normalized as $S^2 = -1$. Taking the matrix element of (16) with the mass term (21) we arrive at formulas known as Wandzura–Wilczek relations [18, 30]. For the region $0 < x < 1$ we have

$$S_{\perp\alpha}\left\{g_\mathrm{T}(x) - \int_x^1 \frac{\mathrm{d}y}{y}\,g_1(y)\right.$$

$$\left. -\frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 \frac{\mathrm{d}y}{y^2}\,h_1(y)\right]\right\} \qquad (29)$$

$$= \frac{1}{2M}\int_x^1 \frac{\mathrm{d}y}{y}\int\limits_{-\infty}^{+\infty}\frac{\mathrm{d}\lambda}{2\pi}\,\mathrm{e}^{-\mathrm{i}\lambda y}$$

$$\times \left\langle P,S_\perp \left| \mathfrak{G}_{5\alpha}\left(\frac{\lambda n}{2}\right)\right| P,S_\perp\right\rangle.$$

An analogous equation for the region $-1 < x < 0$ reads

$$S_{\perp\alpha}\left\{g_\mathrm{T}(x) + \int_{-1}^x \frac{\mathrm{d}y}{y}\,g_1(y)\right.$$

$$\left. -\frac{m}{M}\left[\frac{h_1(x)}{x} + \int_{-1}^x \frac{\mathrm{d}y}{y^2}\,h_1(y)\right]\right\} \qquad (30)$$

$$= -\frac{1}{2M}\int_{-1}^x \frac{\mathrm{d}y}{y}\int\limits_{-\infty}^{+\infty}\frac{\mathrm{d}\lambda}{2\pi}\,\mathrm{e}^{-\mathrm{i}\lambda y}$$

$$\times \left\langle P,S_\perp \left| \mathfrak{G}_{5\alpha}\left(\frac{\lambda n}{2}\right)\right| P,S_\perp\right\rangle.$$

It is easy to get the Wandzura–Wilczek relations for the Mellin moments. For example, in the massless case they have the form

$$S_{\perp\alpha}\int_{-1}^1 \mathrm{d}x\,x^m\left[g_\mathrm{T}(x) - \frac{1}{m+1}g_1(x)\right] \qquad (31)$$

$$= \frac{(-\mathrm{i})^m}{2M(m+1)}\frac{\partial^m}{\partial\lambda^m}\left\langle P,S_\perp \left| \mathfrak{G}_{5\alpha}\left(\frac{\lambda n}{2}\right)\right| P,S_\perp\right\rangle\bigg|_{\lambda=0}.$$

In particular, for $m = 0$ we have

$$\int_{-1}^1 \mathrm{d}x\,[g_\mathrm{T}(x) - g_1(x)] = 0; \qquad (32)$$

for $m = 1$ we get

$$\int_{-1}^1 \mathrm{d}x\,x\left[g_\mathrm{T}(x) - \frac{1}{2}g_1(x)\right] = 0; \qquad (33)$$

and for $m = 2$ we arrive at

$$S_{\perp\alpha}\int_{-1}^1 \mathrm{d}x\,x^2\left[g_\mathrm{T}(x) - \frac{1}{3}g_1(x)\right] \qquad (34)$$

$$= \frac{1}{6M}\left\langle P,S_\perp \left| g\,\bar{\psi}\left[\widetilde{G}_{\alpha\beta}n^\beta\hat{n} - n_\alpha\widetilde{G}_{\lambda\beta}n^\beta\gamma^\lambda\right]\psi\right| P,S_\perp\right\rangle.$$

The second term in (34) is purely longitudinal in α and does not give any contribution; therefore the transverse part of (34) takes the form

$$S_{\perp\alpha}\int_{-1}^1 \mathrm{d}x\,x^2\left[g_\mathrm{T}(x) - \frac{1}{3}g_1(x)\right] \qquad (35)$$

$$= \frac{1}{6M}\left\langle P,S_\perp \left| g\,\bar{\psi}\widetilde{G}_{\alpha\beta}n^\beta\hat{n}\psi\right| P,S_\perp\right\rangle, \qquad \alpha = 1,2.$$

The validity of (32)–(34) follows from the observation that $\mathfrak{G}_5(\lambda n/2) \sim \lambda^2$ and, consequently, the first non-vanishing moment in (31) is the x^2-moment. Equations (32) and (33) are known as the Burkhardt–Cottingham [31] and Efremov–Leader–Teryaev [32] sum rules, respectively. Using the definition of the constant $d^{(2)}$, which measures the effect of genuine twist-3 contributions:

$$\int_{-1}^1 \mathrm{d}x\,x^2\left[g_\mathrm{T}(x) - \frac{1}{3}g_1(x)\right] = \frac{2}{3}\,d^{(2)}, \qquad (36)$$

we come to the identity

$$S_\perp^\alpha\left\langle P,S_\perp \left| g\,\bar{\psi}\widetilde{G}_{\alpha\beta}n^\beta\hat{n}\psi\right| P,S_\perp\right\rangle = -4Md^{(2)}, \qquad (37)$$

which illustrates that the constant $d^{(2)}$ characterizes the correlations of quarks and gluons in a hadron.

4 Non-diagonal nucleon matrix elements

In this section we would like to concentrate on the gluon part $V_\alpha^{\text{tw3}}(x, -x)$ of (24)–(26) with $x = \frac{1}{2}\lambda n$. The Fourier transformation of its non-diagonal nucleon matrix element

$$\mathcal{F}_\alpha^{\text{tw3}}(P', S'; P, S; x) \tag{38}$$

$$= \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \left\langle P', S' \left| V_\alpha^{\text{tw3}}\left(\frac{\lambda n}{2}, -\frac{\lambda n}{2}\right) \right| P, S \right\rangle$$

is the gluon part of the vector GPD, where the spin 4-vectors are normalized as $S^2 = S'^2 = -1$. The Mellin moments of $\mathcal{F}_\alpha^{\text{tw3}}$ are given by

$$\int_{-1}^{1} dx\, x^m \mathcal{F}_\alpha^{\text{tw3}}(P', S'; P, S; x) \tag{39}$$

$$= (-i)^m \frac{\partial^m}{\partial\lambda^m} \left\langle P', S' \left| V_\alpha^{\text{tw3}}\left(\frac{\lambda n}{2}, -\frac{\lambda n}{2}\right) \right| P, S \right\rangle \bigg|_{\lambda=0}.$$

It is not difficult to predict the results of the calculation of the first moments in (39). The $\mathfrak{G}$-term in (26) is proportional to x^2 (17) and, consequently, to λ^2. The first and the third terms in (26) are proportional to x^3 and, consequently, to λ^3. This means that the x^0- and x^1-moments should vanish; the $\mathfrak{G}$-term defines the x^2-moment, with the other terms of (26) giving their contributions to the highest moments only. Thus one can get by direct calculation that

$$\int_{-1}^{1} dx\, \mathcal{F}_\alpha^{\text{tw3}}(x; P', S'; P, S)$$

$$= \int_{-1}^{1} dx\, x \mathcal{F}_\alpha^{\text{tw3}}(x; P', S'; P, S) = 0. \tag{40}$$

These results mean that the first two moments of the GPDs do not get gluon contributions, the WW approximation turns out to be exact for them.

The first non-trivial Mellin moment sensitive to the genuine twist-3 contributions is the x^2-moment. It has the following form:

$$\int_{-1}^{1} dx\, x^2 \mathcal{F}_\alpha^{\text{tw3}}(x; P', S'; P, S) \tag{41}$$

$$= \frac{1}{3}\left\langle P', S' \left| g\bar{\psi}\left[\widetilde{G}_{\alpha\beta}n^\beta\hat{n} - n_\alpha\widetilde{G}_{\lambda\beta}n^\beta\gamma^\lambda\right]\gamma_5\psi \right| P, S \right\rangle.$$

The transverse part of (41) has the form

$$\int_{-1}^{1} dx\, x^2 \mathcal{F}_\alpha^{\text{tw3}}(x; P', S'; P, S) \tag{42}$$

$$= \frac{1}{3}\left\langle P', S' \left| g\bar{\psi}\widetilde{G}_{\alpha\beta}n^\beta\hat{n}\gamma_5\psi \right| P, S \right\rangle, \quad \alpha = 1, 2.$$

The expressions for the axial moments can be obtained using the substitution: $\Gamma\psi \longrightarrow \Gamma\gamma_5\psi$.

5 Estimate of the nucleon matrix elements

In order to estimate the magnitude of the second moments (35) and (42), we should analyze the non-diagonal *hadron* matrix elements of the quark–gluon operators:

$$O_\perp = g\bar{\psi}\widetilde{G}_{\perp\lambda}n^\lambda\hat{n}\psi, \tag{43}$$

$$O_{5\perp} = g\bar{\psi}\widetilde{G}_{\perp\lambda}n^\lambda\hat{n}\gamma_5\psi \tag{44}$$

(in this section we will use the notation: $O_\perp \equiv O_\alpha$, $\alpha = 1, 2$). For this purpose we first consider the non-diagonal *quark* matrix elements of the operators (43) and (44). A method for the calculation of such matrix elements in the model of the instanton vacuum was suggested in [23,24]. It was developed in the framework of the effective chiral theory of the nucleon [33] (see also the recent reviews in [34,35] and the references therein). The essence of the method is in the following.

In the model of the instanton vacuum [22] the fermion partition function in the presence of N_+ instantons and N_- anti-instantons can be represented as a functional integral over the fermion fields with a fermion effective action $S_{\text{eff}}[\bar{\psi}, \psi]$:

$$Z_{N_\pm}^{\text{fermions}} = Z_{N_\pm} C \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{iS_{\text{eff}}[\bar{\psi}, \psi]\right\}, \tag{45}$$

where in the case of one quark flavor

$$S_{\text{eff}}[\bar{\psi}, \psi] = \int \frac{d^4k}{(2\pi)^4}\, \bar{\psi}(k)\left[\hat{k} - M(ik)\right]\psi(k), \tag{46}$$

and $Z_{N_\pm}$ is the variational partition function of the gluodynamics. Equation (46) points out that a quark propagating in the instanton medium acquires a dynamical momentum-dependent mass $M(ik)$. It is this mass that leads to the non-vanishing value of the vacuum condensate $\langle\bar{\psi}\psi\rangle$ and, consequently, to the spontaneous chiral symmetry breaking in QCD [34]. The quark propagator in the instanton vacuum model can be written in the momentum representation as

$$\hat{S}(k) = i\frac{\hat{k} + M(ik)}{k^2 - M^2(ik) + i\varepsilon}. \tag{47}$$

The dynamical mass $M(ik)$ has the form

$$M(ik) = M|F(ik)|^2. \tag{48}$$

Here the mass $M \sim \rho/R^2 \sim 345\,\text{MeV}$, where ρ is the average size of the instantons in the instanton medium and R is the average distance between instantons. The instanton medium is characterized by its "packing fraction" ρ/R with a phenomenological value $\rho/R \sim 1/3$. Thus, the product $M\rho \sim \rho^2/R^2$ is parametrically small. The function $F(k)$ in (48) is a form factor proportional to the Fourier transformation of the wave function of the fermion zero-mode:

$$F(k) = -t\frac{d}{dt}\left[I_0(t)K_0(t) - I_1(t)K_1(t)\right], \quad t = \frac{1}{2}k\rho,$$

$$F(k) = 1, \quad \rho \to 0, \tag{49}$$

where $I_n(t)$ are the modified Bessel functions and $K_n(t)$ are the modified Hankel functions.

The effective action (46) can be used in the calculation of the quark matrix elements of the quark–gluon operators (43) and (44), the operators being replaced by the effective ones:

$$\langle p', s' |O| p, s\rangle_{\text{inst.vac.}} = \langle p', s' |\text{``}O\text{''}| p, s\rangle_{\text{eff}}. \qquad (50)$$

Here "O" is the *fermion* operator, which represents the *quark–gluon* operator O in the effective fermion theory. The explicit form of "O" was obtained in [23, 36]. The effective action and effective operators being known, it is possible to calculate the quark matrix elements of interest.

The direct calculation for the case of the operators (43) and (44) results in the following expressions:

$$\langle p', s' |\text{``}O_\perp\text{''}| p, s\rangle_{\text{eff}} = \frac{1}{2} M \rho^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \, G(\mathrm{i}k)$$

$$\times \bar{u}(p', s')$$

$$\times \Big[F^*(\mathrm{i}(k-p))F(\mathrm{i}p) \, \hat{n}\gamma_5 \hat{S}(k-p)\hat{\Sigma}_\perp(k, n)$$

$$- F^*(\mathrm{i}p')F(\mathrm{i}(k-p')) \, \hat{\Sigma}_\perp(k, n)\hat{S}(k-p')\hat{n}\gamma_5 \Big]$$

$$\times u(p, s)$$

$$+ (\text{corrections}), \qquad (51)$$

$$\langle p', s' |\text{``}O_{5\perp}\text{''}| p, s\rangle_{\text{eff}} = \frac{1}{2} M \rho^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \, G(\mathrm{i}k)$$

$$\times \bar{u}(p', s')$$

$$\times \Big[F^*(\mathrm{i}(k-p))F(\mathrm{i}p) \, \hat{n}\hat{S}(k-p)\hat{\Sigma}_\perp(k, n)$$

$$- F^*(\mathrm{i}p')F(\mathrm{i}(k-p')) \, \hat{\Sigma}_\perp(k, n)\hat{S}(k-p')\hat{n} \Big]$$

$$\times u(p, s)$$

$$+ (\text{corrections}). \qquad (52)$$

Here $G(k)$ is the form factor proportional to the Fourier transformation of the one-instanton dual field strength

$$G(k)$$
$$= 32\pi^2 \left[\left(\frac{1}{2} + \frac{4}{t^2} \right) K_0(t) + \left(\frac{2}{t} + \frac{8}{t^3} \right) K_1(t) - \frac{8}{t^4} \right],$$
$$t = k\rho, \qquad (53)$$
$$G(k) = -4\pi^2, \quad \rho \to 0, \qquad (54)$$

and the following definitions have been used:

$$\hat{\Sigma}_\alpha(k, n) = \frac{k_\alpha k^\mu}{k^2} n^\nu \sigma_{\mu\nu} + \frac{k^\nu k^\lambda}{k^2} n_\lambda \sigma_{\alpha\nu} - \frac{1}{2} n^\nu \sigma_{\alpha\nu},$$

$$\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu], \qquad n^\lambda = \frac{1}{\bar{p}_+ \sqrt{2}}(1; 0, 0, -1),$$

where $u(p, s)$ is the quark Dirac bispinor.

Using the equations of motion, (51) and (52) can be reduced to the form

$$\langle p', s' |\text{``}O_\perp\text{''}| p, s\rangle_{\text{eff}} = M I(p) \, \bar{u}(p', s') \, \hat{n}\gamma_\perp\gamma_5 \, u(p, s), \qquad (55)$$

$$\langle p', s' |\text{``}O_{5\perp}\text{''}| p, s\rangle_{\text{eff}} = -\xi M I(p) \, \bar{u}(p', s') \, \hat{n}\gamma_\perp \, u(p, s), \qquad (56)$$

where $\xi = -\frac{1}{2}(p' - p) \cdot n$, and the integral $I(p)$ is defined in leading order of ρ^2/R^2 as

$$I(p) = \rho^2 \frac{1}{3} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{G(k)F^3(k-p)}{(k-p)^2 + M^2 F^2(k-p)}$$

$$\times \left[-1 + 4\frac{(k \cdot p)^2}{k^2 p^2} \right], \qquad (57)$$

where the integration is performed in Euclidean space with $p^2 = -M^2$. In the case of diagonal matrix elements we reproduce (up to a sign) the results of [24]:

$$\langle p, s |\text{``}O_\perp\text{''}| p, s\rangle_{\text{eff}} = 4M d^{(2)}_{\text{quark}} s_\perp, \qquad (58)$$

$$\langle p, s |\text{``}O_{5\perp}\text{''}| p, s\rangle_{\text{eff}} = 0, \qquad (59)$$

where

$$d^{(2)}_{\text{quark}} = \frac{1}{2} I(p) \qquad (60)$$

is a quark constant analogous to the hadron constant (36) in (37). Thus, the non-diagonal quark matrix elements (55) and (56) of the twist-3 quark–gluon operators can be expressed through the quark constant:

$$\langle p', s' |\text{``}O_\perp\text{''}| p, s\rangle_{\text{eff}}$$
$$= 2M d^{(2)}_{\text{quark}} \, \bar{u}(p', s') \, \hat{n}\gamma_\perp\gamma_5 \, u(p, s), \qquad (61)$$

$$\langle p', s' |\text{``}O_{5\perp}\text{''}| p, s\rangle_{\text{eff}}$$
$$= -2\xi M d^{(2)}_{\text{quark}} \, \bar{u}(p', s') \, \hat{n}\gamma_\perp \, u(p, s). \qquad (62)$$

In the limit of a small average size of instantons ρ we get the following estimate of the integral $I(p)$; see (57):

$$d^{(2)}_{\text{quark}} \sim I(p) \sim \rho^2 M^2 \ln \rho^2 M^2, \qquad \rho \ll R. \qquad (63)$$

The quantity $M\rho \sim \pi\rho^2/R^2$ is a small parameter in the effective fermion theory and, thus, we notice that the quark matrix elements of the quark–gluon operators are parametrically suppressed as a fourth power of the "packing fraction" ρ/R of the instanton vacuum. The numerical calculation [24] of $I(p)$ gives $d^{(2)}_{\text{quark}} = 0.011$ at the phenomenological value $M\rho = 0.58$.

The non-zero value of $d^{(2)}_{\text{quark}}$ appears only at the order $\sim \pi^2 \rho^4/R^4$. At the present stage of development of our theory we cannot make precise predictions at this order, as this would require one to go beyond the one-instanton approximation. Therefore we consider the value of $d^{(2)}_{\text{quark}} = 0.011$ only as an order of magnitude estimate. Using this

order of magnitude estimate we can predict the order of magnitude of $d^{(2)}$ for the nucleon [24]:

$$d^{(2)} \sim 10^{-3}, \tag{64}$$

with

$$d_p^{(2)} - d_n^{(2)} \sim 1/N_c, \quad d_p^{(2)} + d_n^{(2)} \sim 1/N_c^2.$$

Without the suppression due to the instanton packing fraction a "natural" value of the nucleon $d^{(2)}$ would be of order $\sim \int_0^1 \mathrm{d}x \; x^2 \; g_1(x) \sim$ few $\times 10^{-2}$. A recent experiment at SLAC [19] quotes the values $d_p^{(2)} = (3.2 \pm 1.7) \cdot 10^{-3}$ and $d_n^{(2)} = (7.9 \pm 4.8) \cdot 10^{-3}$ at $Q^2 \approx 5$ GeV^2. These data hint to a possible "anomalous suppression" of $d^{(2)}$ as predicted in the model of the instanton vacuum [24].

Using the results (61) and (62) for the quark matrix elements we can make a simple estimate of the corresponding nucleon matrix element at $\Delta^2 = 0$. In such a kind of estimate we assume that the nucleon matrix element at $\Delta^2 = 0$ has the same structure as in (61) and (62). Such an assumption corresponds to the use of a simple quark model to "translate" the quark matrix elements to the nucleon ones. More sophisticated estimates can be done with the help of the chiral quark-soliton model; see e.g. [24,37]. Note, however, that for the very fact of the suppression of the twist-3 nucleon matrix elements the usage of the simple quark model is sufficient. In the simple quark model of the nucleon we arrive at

$$\langle P', S' | \text{“}O_\perp\text{”} | P, S \rangle_{\text{eff}}$$
$$= 2 M_N d^{(2)} \; \bar{N}(P', S') \; \hat{n}\gamma_\perp\gamma_5 \; N(P, S), \tag{65}$$

$$\langle P', S' | \text{“}O_{5\perp}\text{”} | P, S \rangle_{\text{eff}}$$
$$= -2\xi M_N d^{(2)} \; \bar{N}(P', S') \; \hat{n}\gamma_\perp \; N(P, S), \tag{66}$$

where $N(P, S)$ is the nucleon Dirac bispinor. We come to the conclusion that the hadron matrix elements of the "genuine twist-3" (quark–gluon) operators are parametrically suppressed in the effective chiral theory by the "packing fraction" of the instanton vacuum.

6 GPDs decomposition

The quark twist-2 (WW) contributions to the nucleon GPD $\mathcal{F}_\perp$ and $\widetilde{\mathcal{F}}_\perp$ have been obtained in [13, 16]. In this section we would like to compare the WW contributions with the genuine twist-3 ones, calculated in the previous section (to be precise, we will compare their x^2-moments). Following [12] we present the "transverse" part of the GPDs in the form[1]:

$$\mathcal{F}_{\perp\mu}(x, \xi, \Delta) = \bar{N}(P', S')$$
$$\times \left\{ (H + E) \; \gamma_\mu^\perp \right.$$

---

[1] We slightly changed the notation of [12] and added supplementary functions which were missing in [12]; see also [38].

$$\left. + G_1 \; \frac{\Delta_\mu^\perp}{2M} + G_2 \; \gamma_\mu^\perp + G_3 \; \Delta_\mu^\perp \hat{n} + G_4 \; \mathrm{i}\varepsilon_{\mu\nu}^\perp \Delta_\perp^\nu \hat{n}\gamma_5 \right\}$$
$$\times N(P, S), \tag{67}$$

$$\widetilde{\mathcal{F}}_{\perp\mu}(x, \xi, \Delta) = \bar{N}(P', S')$$
$$\times \left\{ \widetilde{H} \; \gamma_\mu^\perp \gamma_5 + \widetilde{E} \; \frac{\Delta_\mu^\perp}{2M}\gamma_5 \right.$$
$$\left. + \widetilde{G}_1 \; \frac{\Delta_\mu^\perp}{2M}\gamma_5 + \widetilde{G}_2 \; \gamma_\mu^\perp \gamma_5 + \widetilde{G}_3 \; \Delta_\mu^\perp \hat{n}\gamma_5 + \widetilde{G}_4 \; \mathrm{i}\varepsilon_{\mu\nu}^\perp \Delta_\perp^\nu \hat{n} \right\}$$
$$\times N(P, S). \tag{68}$$

Note that in the above equations all functions depend on x, ξ and Δ^2. The first three moments of the quark contributions have the following structure.

The x^0-moments are

$$\int_{-1}^1 \mathrm{d}x \, \mathcal{F}_{\perp\mu}(x, \xi, \Delta)$$
$$= \bar{N}(P', S')\{G_M(\Delta^2) \; [\gamma_\mu^\perp]\} N(P, S), \tag{69}$$

$$\int_{-1}^1 \mathrm{d}x \, \widetilde{\mathcal{F}}_{\perp\mu}(x, \xi, \Delta)$$
$$= \bar{N}(P', S') \left\{ G_P(\Delta^2) \; \left[\frac{\Delta_\mu^\perp}{2M}\gamma_5 \right] + G_A(\Delta^2) \; [\gamma_\mu^\perp \gamma_5] \right\}$$
$$\times N(P, S). \tag{70}$$

Here the well-known sum rules [11] have been used:

$$\int_{-1}^1 \mathrm{d}x \, H(x, \xi, \Delta^2) = F_1(\Delta^2),$$
$$\int_{-1}^1 \mathrm{d}x \, E(x, \xi, \Delta^2) = F_2(\Delta^2),$$
$$\int_{-1}^1 \mathrm{d}x \, \widetilde{H}(x, \xi, \Delta^2) = G_A(\Delta^2),$$
$$\int_{-1}^1 \mathrm{d}x \, \widetilde{E}(x, \xi, \Delta^2) = G_P(\Delta^2), \tag{71}$$

where $F_1(\Delta^2)$ and $F_2(\Delta^2)$ are the Dirac and Pauli form factors, $G_P(\Delta^2)$ and $G_A(\Delta^2)$ are the pseudo-scalar and axial-vector form factors, and, by definition, the magnetic form factor is

$$G_M(\Delta^2) = F_1(\Delta^2) + F_2(\Delta^2). \tag{72}$$

The results (69) and (70) imply that

$$\int_{-1}^1 \mathrm{d}x \, G_i(x, \xi, \Delta) = 0, \qquad \int_{-1}^1 \mathrm{d}x \, \widetilde{G}_i(x, \xi, \Delta) = 0.$$

These sum rules can be considered as a non-forward generalization of the Burkhardt–Cottingham [31] sum rules (32).

For the x-moments of the vector GPD we get

$$\int_{-1}^{1} \mathrm{d}x\, x\, G_1(x,\xi,\Delta) = \frac{1}{2}\frac{\partial}{\partial\xi}\int_{-1}^{1} \mathrm{d}x\, x\, E(x,\xi,\Delta), \quad (73)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, G_2(x,\xi,\Delta)$$
$$= \frac{1}{2}\left[G_A(\Delta^2) - \int_{-1}^{1} \mathrm{d}x\, x\,(H+E)(x,\xi,\Delta)\right], \quad (74)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, G_3(x,\xi,\Delta) = 0, \quad (75)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, G_4(x,\xi,\Delta) = 0. \quad (76)$$

Note that the x-moment of the GPD G_2 does not vanish in the forward limit and gives the quark orbital momentum of the nucleon [12]:

$$\lim_{\Delta\to 0}\int_{-1}^{1} \mathrm{d}x\, x\, G_2(x,\xi,\Delta) = -J_q + \frac{1}{2}\,\Delta q = -L_q\,.$$

For the x-moments of the axial-vector GPD we have

$$\int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{G}_1(x,\xi,\Delta)$$
$$= \frac{1}{2}\left[F_2(\Delta^2) + \left(\xi\frac{\partial}{\partial\xi} - 1\right)\int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{E}(x,\xi,\Delta)\right], \quad (77)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{G}_2(x,\xi,\Delta) \quad (78)$$
$$= \frac{1}{2}\left[\xi^2 G_E(\Delta^2) - \frac{\Delta^2}{4M^2}F_2(\Delta^2) - \int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{H}(x,\xi,\Delta)\right],$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{G}_3(x,\xi,\Delta) = \frac{1}{4}\,\xi\, G_E(\Delta^2), \quad (79)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, \widetilde{G}_4(x,\xi,\Delta) = \frac{1}{4}\,G_E(\Delta^2), \quad (80)$$

where

$$G_E(\Delta^2) = F_1(\Delta^2) + \frac{\Delta^2}{4M^2}\,F_2(\Delta^2) \quad (81)$$

is the electric form factor. These moments have no gluon contribution due to (40) and they have already been discussed in [12, 16]. Similarly to the previous case one can consider the relations (73)–(76) and (77)–(80) as a non-forward generalization of the Efremov–Leader–Teryaev [32] sum rule (33).

We are passing now to a consideration of the x^2-moments. For these moments the "genuine twist-3" (quark–gluon) contribution (42) does not vanish and one can decompose the GPDs G and $\widetilde{G}$ from (67) and (68) into the WW and genuine twist-3 parts:

$$G_i^{(2)}(\xi,\Delta) = G_i^{(2)\mathrm{WW}}(\xi,\Delta) + G_i^{(2)\mathrm{tw3}}(\xi,\Delta),\ i = 1,2,3,4. \quad (82)$$

For the x^2-moments of the vector GPD "kinematical" part we get

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, G_1^{\mathrm{WW}}(x,\xi,\Delta)$$
$$= \left(-\frac{1}{3}\right)\left[\xi F_2(\Delta^2) + \xi\left(\xi\frac{\partial}{\partial\xi} - 1\right)\widetilde{E}^{(1)}(\xi,\Delta)\right.$$
$$\left. - \frac{\partial}{\partial\xi}E^{(2)}(\xi,\Delta)\right], \quad (83)$$

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, G_2^{\mathrm{WW}}(x,\xi,\Delta)$$
$$= \frac{1}{3}\left[\xi^2 G_M(\Delta^2) + \frac{\Delta_\perp^2}{4M^2}\left(\xi\frac{\partial}{\partial\xi} - 1\right)\widetilde{E}^{(1)}(\xi,\Delta)\right.$$
$$\left. + \widetilde{H}^{(1)}(\xi,\Delta) - 2(H+E)^{(2)}(\xi,\Delta)\right], \quad (84)$$

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, G_3^{\mathrm{WW}}(x,\xi,\Delta)$$
$$= \frac{1}{6}\left[\xi G_M(\Delta^2) + \left(1 - \frac{\Delta^2}{4M^2}\right)\xi\left(\xi\frac{\partial}{\partial\xi} - 1\right)\widetilde{E}^{(1)}(\xi,\Delta)\right.$$
$$\left. - \frac{\partial}{\partial\xi}(H+E)^{(2)}(\xi,\Delta)\right], \quad (85)$$

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, G_4^{\mathrm{WW}}(x,\xi,\Delta) \quad (86)$$
$$= \left(-\frac{1}{6}\right)\left(\xi\frac{\partial}{\partial\xi} - 1\right)\left[\widetilde{H}^{(1)}(\xi,\Delta) + \frac{\Delta^2}{4M^2}\widetilde{E}^{(1)}(\xi,\Delta)\right],$$

where we used shorthand notation for the Mellin moments of the twist-2 vector GPDs:

$$H^{(m)}(\xi,\Delta) = \int_{-1}^{1} \mathrm{d}x\, x^m\, H(x,\xi,\Delta),$$

$$E^{(m)}(\xi,\Delta) = \int_{-1}^{1} \mathrm{d}x\, x^m\, E(x,\xi,\Delta);$$

and analogous notation for the twist-2 pseudo-vector GPDs $\widetilde{H}$ and $\widetilde{E}$.

For the x^2-moments of the axial-vector GPD we get

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, \widetilde{G}_1^{\mathrm{WW}}(x,\xi,\Delta)$$
$$= \left(-\frac{1}{3}\right)\left(\xi\frac{\partial}{\partial\xi} - 2\right)\left[E^{(1)}(\xi,\Delta) - \widetilde{E}^{(2)}(\xi,\Delta)\right], \quad (87)$$

$$\int_{-1}^{1} \mathrm{d}x\, x^2\, \widetilde{G}_2^{\mathrm{WW}}(x,\xi,\Delta)$$
$$= \frac{1}{3}\left[\xi^2\left(G_A(\Delta^2) + (H+E)^{(1)}(\xi,\Delta)\right)\right. \quad (88)$$
$$\left. + \frac{\Delta_\perp^2}{4M^2}\left(\xi\frac{\partial}{\partial\xi} - 2\right)E^{(1)}(\xi,\Delta) - 2\widetilde{H}^{(2)}(\xi,\Delta)\right],$$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_3^{\mathrm{WW}}(x, \xi, \Delta)$$

$$= \frac{1}{6}\left[\xi\left(G_A(\Delta^2) + (H+E)^{(1)}(\xi, \Delta) \right) \right.$$

$$+ \left(1 - \frac{\Delta^2}{4M^2}\right)\xi\left(\xi\frac{\partial}{\partial\xi} - 2\right)E^{(1)}(\xi, \Delta)$$

$$\left. - \frac{\partial}{\partial\xi}\widetilde{H}^{(2)}(\xi, \Delta) \right], \tag{89}$$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_4^{\mathrm{WW}}(x, \xi, \Delta) \tag{90}$$

$$= \left(-\frac{1}{6}\right)\left(\xi\frac{\partial}{\partial\xi} - 2\right)\left[H^{(1)}(\xi, \Delta) + \frac{\Delta^2}{4M^2}E^{(1)}(\xi, \Delta) \right].$$

Let us consider the quark–gluon contributions to the moments of $\mathcal{F}_\perp$ and $\widetilde{\mathcal{F}}_\perp$. We have already shown that their x^0- and x^1-moments vanish (40), with the x^2-moment being given by (42) in the vector case. Substituting (65) into (42) and decomposing the result according to (67) we get the following expressions for the x^2-moments of the genuine twist-3 vector GPD at $\Delta^2 = 0$:

$$\int_{-1}^{1} dx\, x^2\, G_1^{\mathrm{tw3}}(x, \xi) = 0, \tag{91}$$

$$\int_{-1}^{1} dx\, x^2\, G_2^{\mathrm{tw3}}(x, \xi) = -\frac{2}{3}\left(1 - \xi^2\right)d^{(2)}, \tag{92}$$

$$\int_{-1}^{1} dx\, x^2\, G_3^{\mathrm{tw3}}(x, \xi) = \frac{1}{3}\,\xi\, d^{(2)}, \tag{93}$$

$$\int_{-1}^{1} dx\, x^2\, G_4^{\mathrm{tw3}}(x, \xi) = \frac{1}{3}\, d^{(2)}. \tag{94}$$

Here $d^{(2)}$ is given by (36). For the axial-vector GPD we get at $\Delta^2 = 0$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_1^{\mathrm{tw3}}(x, \xi) = 0, \tag{95}$$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_2^{\mathrm{tw3}}(x, \xi) = \frac{2}{3}\left(1 - \xi^2\right)d^{(2)}, \tag{96}$$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_3^{\mathrm{tw3}}(x, \xi) = -\frac{1}{3}\,\xi\, d^{(2)}, \tag{97}$$

$$\int_{-1}^{1} dx\, x^2\, \widetilde{G}_4^{\mathrm{tw3}}(x, \xi) = -\frac{1}{3}\, d^{(2)}. \tag{98}$$

The expressions (91)–(94) and (95)–(98) for the genuine twist-3 contributions should be compared to the expressions (83)–(86) and (87)–(90) of the WW part for the quark contributions, respectively. As we saw above, in the instanton vacuum the constant $d^{(2)}$ is strongly suppressed by the packing fraction of instantons [24]. Here we expressed the x^2-moments of the genuine twist-3 GPDs in terms of the constant $d^{(2)}$; see (91)–(94) and (95)–(98).

We conclude that the twist-3 vector and axial-vector GPDs are parametrically suppressed relative to the WW ones due to the packing fraction of the instanton vacuum:

$$\frac{G_i^{(2)\mathrm{tw3}}}{G_i^{(2)\mathrm{WW}}} \sim \pi^2\left(\frac{\rho^2}{R^2}\right)^2\ln\left(\frac{\rho^2}{R^2}\right) \ll 1. \tag{99}$$

This indicates that in the QCD vacuum the "genuine twist-3" (quark–gluon) contributions turn out to be small relative to the purely quark ones.

References

1. D. Müller et al., Fortschr. Phys. **42**, 101 (1994)
2. X. Ji, Phys. Rev. D **55**, 7114 (1997)
3. A.V. Belitsky, D. Müller, A. Kirchner, Nucl. Phys. B **629**, 323 (2002)
4. C. Adloff et al. [H1 Collaboration], Phys. Lett. B **517**, 47 (2001)
5. A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. **87**, 182001 (2001)
6. S. Stepanyan et al. [CLAS Collaboration], Phys. Rev. Lett. **87**, 182002 (2001)
7. F. Ellinghaus [for the HERMES Collaboration], Nucl. Phys. A **711**, 171 (2002)
8. X. Ji, J. Phys. G **24**, 1181 (1998)
9. A.V. Radyushkin, in At the Frontier of Particle Physics, edited by M. Shifman (World Scientific, Singapore 2001)
10. K. Goeke, M.V. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. **47**, 401 (2001)
11. X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
12. M. Penttinen, M.V. Polyakov, A.G. Schuvaev, M.I. Strikman, Phys. Lett. B **491**, 96 (2000)
13. A.V. Belitsky, D. Müller, Nucl. Phys. B **589**, 611 (2000)
14. I.V. Anikin, B. Pire, O.V. Teryaev, Phys. Rev. D **62**, 071501 (2000)
15. A.V. Radyushkin, C. Weiss, Phys. Rev. D **63**, 114012 (2001)
16. N. Kivel, M.V. Polyakov, Nucl. Phys. B **600**, 334 (2001)
17. N. Kivel, M.V. Polyakov, A. Schäfer, O.V. Teryaev, Phys. Lett. B **497**, 73 (2001)
18. S. Wandzura, F. Wilczek, Phys. Lett. B **72**, 195 (1977)
19. P.L. Anthony et al. [E155 Collaboration], Phys. Lett. B **553**, 18 (2003)
20. G.S. Mitchell [for the E155 Collaboration], hep-ex/9903055
21. P. Bosted [for the E155x Collaboration], Nucl. Phys. A **663**, 297 (2000); A **666**, 300 (2000)
22. D. Diakonov, V. Petrov, Nucl. Phys. B **245**, 259 (1984); B **272**, 457 (1986)
23. D.I. Diakonov, M.V. Polyakov, C. Weiss, Nucl. Phys. B **461**, 539 (1996)
24. J. Balla, M.V. Polyakov, C. Weiss, Nucl. Phys. B **510**, 327 (1997)

25. B. Dressler, M.V. Polyakov, Phys. Rev. D **61**, 097501 (2000)
26. I.I. Balitsky, Phys. Lett. B **124**, 230 (1983)
27. I.I. Balitsky, V.M. Braun, Nucl. Phys. B **311**, 541 (1988/1989)
28. M. Anselmino, A. Efremov, E. Leader, Phys. Rept. **261**, 1 (1995) [Erratum **281**, 399 (1995)]
29. R.L. Jaffe, in Proceedings of the International School on Nucleon Spin Structure, Erice, Italy, 1995, edited by B. Frois, V.W. Hughes, N. de Groot (World Scientific, 1997), p. 42
30. R.D. Tangerman, P.J. Mulders, hep-ph/9408305
31. H. Burkhardt, W.N. Cottingham, Ann. Phys. **56**, 453 (1970)
32. A.V. Efremov, O.V. Teryaev, E. Leader, Phys. Rev. D **55**, 4307 (1997)
33. D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, Nucl. Phys. B **306**, 809 (1988)
34. D. Diakonov, in Selected Topics in Nonperturbative QCD, Proceedings of Enrico Fermi International School of Physics, Course 80, Varenna, Italy, edited by A. Di Giacomo, D. Diakonov (IOS Press, 1996), p. 397; hep-ph/0212026
35. D. Diakonov, V.Yu. Petrov, in At the Frontier of Particle Physics, edited by M. Shifman (World Scientific, Singapore 2001)
36. M.V. Polyakov, C. Weiss, Phys. Lett. B **387**, 841 (1996)
37. N.Y. Lee, K. Goeke, C. Weiss, Phys. Rev. D **65**, 054008 (2002)
38. A. Mukherjee, M. Vanderhaeghen, hep-ph/0211386